Key.

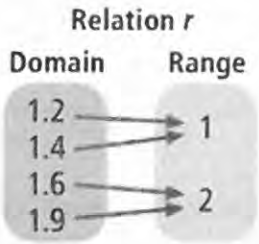
Algebra 2

Notes 5-6 Inverse relations and Functions

ObJ: **Represent the inverse of a relation using tables, graphs, and equations**

<u>What is a function?</u> each element of domain paired to one element of range

<u>What is an inverse?</u> → switch x+y (x, y) → (y, x)



Notation: If the inverse is also a function, then we denote it as f^{-1}

The domain of the original function is the range of the inverse.

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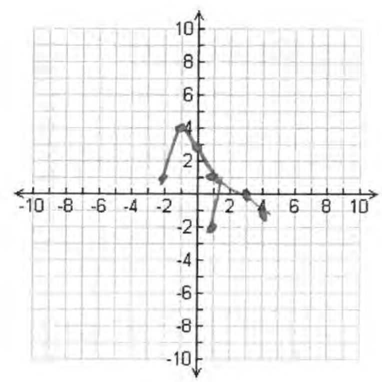
Example 1. Find the inverse. Graph the relation and its inverse.

x	y
−2	1
−1	4
0	3
1	1

inv.

x	y
1	-2
4	-1
3	0
1	1



<u>**Is the inverse a function?**</u> no, x=1 repeats.

You try: Find the inverse and tell if it is a function.

x	−1	0	1	2	3	4
y	9	7	5	3	1	−1

x	9	7	5	3	1	-1
y	-1	0	1	2	3	4

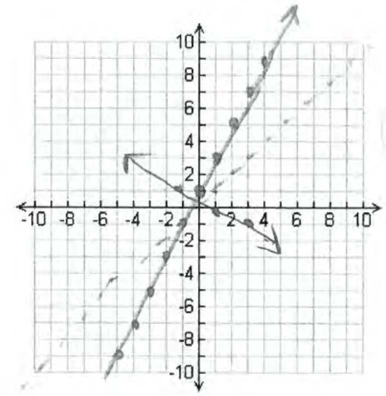
yes no x repeats

Activity

1. Graph y=2x+1
2. Graph y=x as dotted
3. Graph the inverse of y

x	y
0	1
1	3
2	5

x	y
1	0
3	1
5	2



What do you notice about the symmetry of the graphs?

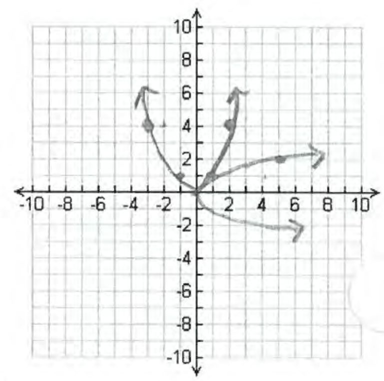
they are symmetric over y=x

Example 3. Find and equation for an inverse.

Given $y = x^2$

Switch x and y. $x = y^2$

Solve for y. $y = \pm\sqrt{x}$



How are the graphs of f(x) and it's inverse related?

symmetric over y=x

You try: f(x)= 2x+1

$x = 2y + 1$

$x - 1 = 2y$

$\frac{x}{2} - \frac{1}{2} = y$

Is the inverse a function?

yes it's linear

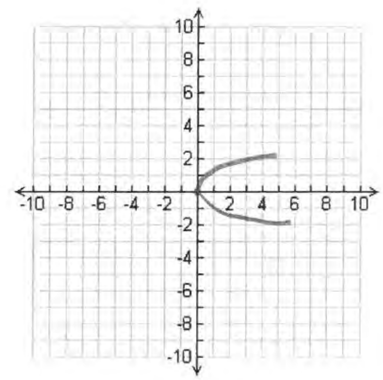
Find $f(x)^{-1}$ Practice.

1. $y = x^3 + 1$
   $x = y^3 + 1$
   $x - 1 = y^3$
   $\sqrt[3]{x-1} = y$

2. $y = 10 - 3x$
   $x = 10 - 3y$
   $x - 10 = -3y$
   $-\frac{1x}{3} + \frac{10}{3} = y$

3. $y = x^2 - 4$
   $x = y^2 - 4$
   $x + 4 = y^2$
   $\pm\sqrt{x+4} = y$

Example 3. Restrict a domain to produce an inverse function.



Given $y = x^2$ $x \geq 0$

$x = y^2$

$y = \pm\sqrt{x}$ → only use Y_2

$y = \sqrt{x}$

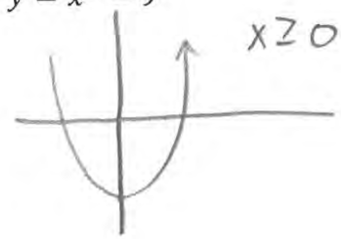
You try. Restrict the domain to find an inverse function. $y = x^2 - 9$

$x = y^2 - 9$

$x + 9 = y^2$

$y = \pm\sqrt{x+9}$

$y = \sqrt{x+9}$ $x \geq -9$



$x \geq 0$

Example 4. Find an equation for and inverse function.

$y = \sqrt{x - 2}$

$x = \sqrt{y-2}$

$x^2 = y - 2$

$x^2 + 2 = y$

Composition of inverses. Two functions are inverses of each other if

$$f \circ f^{-1}(x) = f^{-1} \circ f(x) = x$$

That is their composition both directions results in x.

Show that f(x) = 2x+5 and $f(x)^{-1} = \frac{1}{2}x - \frac{5}{2}$ are inverses of each other.

$f(f^{-1}(x)) = 2(\frac{1}{2}x - \frac{5}{2}) + 5$
$= x - 5 + 5$
$= x$ ✓

$f^{-1}(f(x)) = \frac{1}{2}(2x+5) - 5/2$
$= x + 5/2 - 5/2$
$= x$ ✓

$\therefore$ inverses

You try. Prove if f(x) and g(x) are inverses.

$f(x) = x^2 + 5$

$g(x) = \sqrt{x} - 5$

$f(g(x)) = (\sqrt{x} - 5)^2 + 5$
$= (\sqrt{x} - 5)(\sqrt{x} - 5) + 5$
$= x - 5\sqrt{x} - 5\sqrt{x} + 25 + 5$
$\neq x$

so not inverses.